

WORKBOOK + HOMEWORK

THE CURVE

Discrete math, proofs, and the semester five strangers almost failed —
Don't tell me. Prove it.

Most of the class is failing EECS 203, and Professor Hale says she'll curve the exams. Five strangers thrown together as a study group — Jaylen, who's never failed anything; Bryce, who's secretly failing everything; Anika, who freezes on exams; Marisol, who works overnights; and Yusuf, who shares everything — have to learn logic, proofs, infinity, induction, counting and graphs together. To pass, they have to stop competing, stop lying about where they stand, and prove it.

How to use this workbook

Most of the class is failing EECS 203, and Professor Hale says she'll curve the exams. Five strangers thrown together as a study group — Jaylen, who's never failed anything; Bryce, who's secretly failing everything; Anika, who freezes on exams; Marisol, who works overnights; and Yusuf, who shares everything — have to learn logic, proofs, infinity, induction, counting and graphs together. To pass, they have to stop competing, stop lying about where they stand, and prove it.

Each episode is 5:33–8:41 long (39 minutes for the season). Watch the episode, then work the matching pages: the one sentence, the words, the proofs from the episode walked step by step, the proof template to keep, the common mistakes, and the homework. Answers are at the back — try first.

THE SEASON



EPISODE 1 · 8:41
Knights and Knaves



EPISODE 2 · 7:15
Proof by Contradiction



EPISODE 3 · 5:59
Some Infinities Are Bigger



EPISODE 4 · 6:03
The Next Step



EPISODE 5 · 5:48
Count Me In



EPISODE 6 · 5:33
The Curve

Watch the season: <https://www.youtube.com/playlist?list=PLA7Y3OruxDgg>

THE CAST

Jaylen Carter — 18 · east side of Detroit · first-gen · has never failed anything (the narrator, grown up)

Anika Rao — 18 · Troy · Dad at GM, Mom at Ford · freezes on exams

Yusuf Haddad — 19 · Dearborn · family bakery on Warren Ave · brings bread to everything

Kwame — GSI · PhD year five · powered by cold brew

Bryce Whitmore — 19 · Bloomfield Hills · legacy · "Whitmores don't get C's"

Marisol Vega — 23 · Southwest Detroit · transfer from Washtenaw CC · works overnights

Professor Hale — EECS 203 · legendary · "Don't tell me. Prove it."



EPISODE 1

Knights and Knaves

8:41 · The Curve · a free EECS 203 course from Bot That Lyfe

An if-then is a promise. It's only broken when you do your part and they don't do theirs.

IN THIS EPISODE

- Propositions, truth tables, and the one row that breaks $p \rightarrow q$ (vacuous truth)
- Converse vs. contrapositive — and the Wason card test (A and 7, not A and 4)
- Negating "for every": the opposite of everybody's failing is somebody isn't
- Proof by cases: knights and knaves without knowing everything

WORDS TO KNOW

Proposition

A sentence that is either true or false.

$p \rightarrow q$

If p , then q . False only when p is true and q is false.

Contrapositive

$\neg q \rightarrow \neg p$ — the same promise said backwards (the converse $q \rightarrow p$ is not).

Proof by cases

Check every possibility; if they all agree, you're done.

WHY COMPUTER SCIENCE NEEDS THIS

Every if-statement in every program is an implication, and every circuit in your laptop is built from $\wedge$, $\vee$ and $\neg$. Programmers get the converse wrong constantly ("the test passed, so the code is correct"), and security bugs live in the rows nobody checked. Logic is how you reason about all the cases, not just the ones you tried.

WALK THE PROOF

Knights and knaves, by cases

1. A says "B is a knave." B says "A and C are the same type." Write down both possibilities for A.
2. Case 1, A is a knight: A tells the truth $\rightarrow$ B is a knave $\rightarrow$ B's sentence is false $\rightarrow$ A and C differ $\rightarrow$ C is a knave.
3. Case 2, A is a knave: A lied $\rightarrow$ B is a knight $\rightarrow$ B's sentence is true $\rightarrow$ A and C are the same $\rightarrow$ C is a knave.
4. Every case says C is a knave, so C is a knave. ■ (You never needed to know A.)

KEEP THIS TEMPLATE

THE PROOF FRAMEWORK

GIVEN: what do we know? (write the definitions out)

PROVE: exactly what must be shown — for every case

STRATEGY: direct · contrapositive · contradiction · cases · induction

DERIVATION: every line + its reason

■ did we prove the original claim, every case?

COMMON MISTAKES

- Treating $p \rightarrow q$ as broken when p is false (it's vacuously true)
- Mixing up the converse $q \rightarrow p$ with the contrapositive $\neg q \rightarrow \neg p$
- Negating "everybody fails" as "everybody passes" (it's "somebody doesn't fail")
- Stopping after one case of a proof by cases

Sources. Rosen, K. H. Discrete Mathematics and Its Applications, §1.1–1.4 (logic, quantifiers, knights and knaves) · Wason, P. C. (1968). Reasoning about a rule. Quarterly Journal of Experimental Psychology, 20(3). · Smullyan, R. (1978). What Is the Name of This Book? (knights and knaves)

Homework: Knights and Knaves

Work it on paper, then check the answer key at the back. Every problem is EECS 203 level.

WARM-UPS

1. Which are propositions? (a) "Ann Arbor is in Michigan." (b) "Close the door." (c) " $x + 2 = 5$ " (d) "Every odd number is prime."
2. p = "it's raining", q = "the Diag is wet." Write $p \rightarrow q$, its converse and its contrapositive in words. Which means the same as $p \rightarrow q$?
3. From memory, fill in the truth table for $p \rightarrow q$ and circle the one row that makes it false.

FILL IN THE BLANKS

4. Fill in columns $\neg p$, $\neg q$, $p \rightarrow q$ and $\neg q \rightarrow \neg p$ for all four rows. "The last two columns are _____, so the statements are logically _____."
5. A says "We are both knaves." If A were a knight, A's sentence would be _____ — impossible. So A is a _____, A's sentence is _____, and B is a _____.

WRITE IT YOURSELF

6. A says "At least one of us is a knave." What are A and B? Prove it by cases.
7. Negate in words and symbols: (a) "Every student failed Exam 1." ($\forall s F(s)$) (b) "Some student got a 100." ($\exists s H(s)$)

SPOT THE LIE

8. "If you study, you'll pass. Jaylen passed. So Jaylen studied." Find the bug, then fix it with the contrapositive.

CHALLENGE

9. A: "B and C are both knights." B: "A is a knave." C: "B is a knave." Who is what? Prove it.
10. Rule: "If you're inside the stadium, you have a ticket." Fans: inside, at home, has a ticket, no ticket. Which must you check, and why is this easier than A/K/4/7?



EPISODE 2

Proof by Contradiction

7:15 · The Curve · a free EECS 203 course from Bot That Lyfe

A proof is an argument nobody can wiggle out of. Checking examples is a **feeling**, not a proof.

IN THIS EPISODE

- Why examples aren't proofs ($n^2 + n + 41$)
- Direct proof: odd + odd = even, as pairs and as algebra
- Contrapositive: n^2 even $\rightarrow$ n even
- Contradiction: $\sqrt{2}$ is irrational; squares leave remainder 0 or 1 mod 4

WORDS TO KNOW

Even / odd

$n = 2k$ / $n = 2k + 1$, where k is how many pairs (k can be any integer).

Direct proof

Walk from what you're given to what you want, one justified step at a time.

Proof by contradiction

Pretend the opposite, follow it until the math breaks.

Remainder (mod)

The leftover: $7 \bmod 4 = 3$ because $7 = 4 \cdot 1 + 3$.

WHY COMPUTER SCIENCE NEEDS THIS

Programs are proven correct the same way: a loop invariant is a direct proof, and many security guarantees are proofs by contradiction. Testing a few inputs is the $n^2 + n + 41$ trap — that's how bugs ship.

WALK THE PROOF

$\sqrt{2}$ is not a fraction

1. Pretend $\sqrt{2} = a/b$, already in lowest terms.
2. Square both sides: $a^2 = 2b^2$, so a^2 is even, so a is even (the contrapositive proof). Write $a = 2c$.
3. $4c^2 = 2b^2$, so $b^2 = 2c^2$, so b is even too.
4. Both even contradicts lowest terms. The pretend was wrong: $\sqrt{2}$ is irrational. ■

KEEP THIS TEMPLATE

PROOF BY CONTRADICTION

CLAIM: _____
PRETEND: the opposite is true: ____
FOLLOW: step $\rightarrow$ step $\rightarrow$ step (reasons)
BREAKS: $\nless$ ____ contradicts ____
SO: the pretend was wrong. ■

COMMON MISTAKES

- "I tried a bunch of numbers" — a feeling, not a proof
- Using the converse instead of the contrapositive
- Contradiction proofs that never actually reach a contradiction
- Dividing by something that might be zero

Sources. Rosen, K. H. Discrete Mathematics and Its Applications, §1.7–1.8, §4.1 · Euler (1772): $n^2 + n + 41$

Homework: Proof by Contradiction

Work it on paper, then check the answer key at the back. Every problem is EECS 203 level.

WARM-UPS

1. Write 13 in the form $2k + 1$. What is k ? Now do -5 .
2. "I checked $n = 1$ to 100." Why isn't that a proof?
3. Find $23 \bmod 4$, $36 \bmod 4$ and $49 \bmod 4$.

FILL IN THE BLANKS

4. Prove even + even = even: let $m = 2a$, $n = 2b$. Then $m + n = \underline{\hspace{1cm}} = 2(\underline{\hspace{1cm}})$. ■
5. Prove "if n^2 is odd, then n is odd": contrapositive — if n is $\underline{\hspace{1cm}}$, then n^2 is $\underline{\hspace{1cm}}$. Let $n = 2k$: $n^2 = \underline{\hspace{1cm}} = 2(\underline{\hspace{1cm}})$. ■

WRITE IT YOURSELF

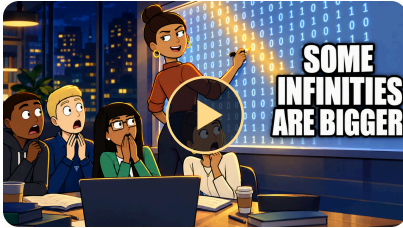
6. Prove: if $3n + 2$ is odd, then n is odd.
7. Prove $\sqrt{3}$ is irrational.

SPOT THE LIE

8. "Let $a = b$. Then $a^2 = ab$, $a^2 - b^2 = ab - b^2$, $(a + b)(a - b) = b(a - b)$, $a + b = b$, $2b = b$, $2 = 1$." Which step is illegal?

CHALLENGE

9. Prove $n^2 + n$ is even for every integer n .
10. Prove no integer x has x^2 leaving remainder 2 when divided by 4.



EPISODE 3

Some Infinities Are Bigger

5:59 · The Curve · a free EECS 203 course from Bot That Lyfe

Two sets are the same size when you can pair them up perfectly, with nobody left out.

IN THIS EPISODE

- Sets, $\cap$ (and) and $\cup$ (or)
- Functions, one-to-one, onto, and bijections
- Hilbert's Hotel and $|\mathbb{N}| = |\mathbb{Z}|$ (the zigzag)
- Cantor's diagonal: infinite binary strings can't be listed

WORDS TO KNOW

Set

A bag of things; order and repeats don't matter.

Bijection

A perfect pairing — one-to-one and onto.

Countable

Can be listed: paired with 0, 1, 2, ...

Uncountable

Too big to list (the reals, infinite 0/1 strings).

WHY COMPUTER SCIENCE NEEDS THIS

Cantor's diagonal is the same trick Turing used to prove the halting problem: no program can decide, for every program, whether it halts. There are more problems than programs. Functions, one-to-one and onto are also everyday CS: hash functions, encodings and database keys are all about who maps to whom.

WALK THE PROOF

Cantor's diagonal

1. Pretend every infinite 0/1 string is in a list $s_1, s_2, s_3, \dots$
2. Build s : flip digit 1 of s_1 , digit 2 of s_2 , digit 3 of s_3 , ...
3. s differs from every s_n in spot n , so it isn't on the list.
4. Contradiction: the strings can't be listed. They're uncountable. ■

KEEP THIS TEMPLATE

SAME SIZE?

1. find a rule f from A to B
2. one-to-one: $f(x) = f(y) \rightarrow x = y$
3. onto: every b in B is $f(\text{something})$
4. both $\rightarrow$ bijection $\rightarrow |A| = |B|$

COMMON MISTAKES

- "A proper subset is smaller" — only for finite sets
- Forgetting to show onto (or one-to-one) — you need both
- Thinking the diagonal string might be on the list somewhere later

Sources. Rosen, K. H. Discrete Mathematics and Its Applications, §2.1–2.5 (sets, functions, cardinality) · Cantor, G. (1891). Über eine elementare Frage der Mannigfaltigkeitslehre. · Hilbert, D. (1924). Lecture on the infinite (the Grand Hotel).

Homework: Some Infinities Are Bigger

Work it on paper, then check the answer key at the back. Every problem is EECS 203 level.

WARM-UPS

1. $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$. Find $A \cup B$, $A \cap B$, $A - B$.
2. Is $\{1, 2, 2, 3\}$ the same set as $\{3, 2, 1\}$?
3. $f(a) = 1$, $f(b) = 1$, $f(c) = 3$ from $\{a, b, c\}$ to $\{1, 2, 3\}$. One-to-one? Onto?

FILL IN THE BLANKS

4. $f: \mathbb{Z} \rightarrow \mathbb{Z}$, $f(n) = 2n + 1$. If $f(m) = f(n)$: $2m + 1 = \underline{\hspace{1cm}}$, so $m = \underline{\hspace{1cm}}$. One-to-one ✓. Onto? Which numbers are missed?
5. Pair $\mathbb{N}$ with the even naturals: $f(n) = \underline{\hspace{1cm}}$. Each $2k$ comes from $n = \underline{\hspace{1cm}}$.

WRITE IT YOURSELF

6. The zigzag $f(n) = (n + 1)/2$ for odd n , $-n/2$ for even n . Compute $f(0) \dots f(6)$ and explain why every integer is hit exactly once.
7. The bakery is full; an infinite bus of new customers arrives. Where does everyone move?

SPOT THE LIE

8. "The even numbers are a proper subset of $\mathbb{N}$, so there are fewer of them."

CHALLENGE

9. Prove the set of all subsets of $\mathbb{N}$ is uncountable.



EPISODE 4

The Next Step

6:03 · The Curve · a free EECS 203 course from Bot That Lyfe

Induction: get on the first step, and prove that from any step you can reach the next one.

IN THIS EPISODE

- Base case + inductive step = every step
- $1 + 2 + \dots + n = n(n + 1)/2$, as a picture and by induction
- Strong induction: postage from 4¢ and 5¢ (four base cases)
- The tromino tiling proof

WORDS TO KNOW

Base case

Show it's true for the first value.

Inductive step

Assume it's true for k ; show it's true for $k + 1$.

Strong induction

Assume it's true for every value up to k .

WHY COMPUTER SCIENCE NEEDS THIS

Induction is recursion seen from the other side: a recursive function works because the base case returns and every call hands off to a smaller one — exactly the tromino proof. Loop invariants, structural induction on trees, and correctness proofs of algorithms all use it.

WALK THE PROOF

Prove $1 + 2 + \dots + n = n(n + 1)/2$

1. Base, $n = 1$: $1 = 1 \cdot 2/2$. ✓
2. Assume $1 + \dots + k = k(k + 1)/2$.
3. Add $k + 1$: $k(k + 1)/2 + (k + 1) = (k + 1)(k + 2)/2$ — the formula for $k + 1$.
4. True at 1, and each step gives the next, so true for all n . ■

KEEP THIS TEMPLATE

INDUCTION

CLAIM: $P(n)$ for every $n \geq ______$

BASE: $P(______)$ is true because $______$

ASSUME: $P(k)$ for some $k \geq ______$

STEP: use $P(k)$ to show $P(k+1)$: $______$

■ base + step $\rightarrow$ every n

COMMON MISTAKES

- Skipping or fumbling the base case
- Assuming $P(k + 1)$ instead of proving it
- Strong induction with too few base cases
- An inductive step that doesn't work for the smallest k (all horses are one color)

Sources. Rosen, K. H. Discrete Mathematics and Its Applications, §5.1–5.2 (induction, strong induction, the tromino tiling) · Golomb, S. W. (1954). Checker boards and polyominoes. American Mathematical Monthly 61.

Homework: The Next Step

Work it on paper, then check the answer key at the back. Every problem is EECS 203 level.

WARM-UPS

1. Use the formula to add $1 + 2 + \cdots + 10$.
2. In the staircase picture, which part is the base case, and which is the inductive step?
3. Make 22¢ from 4¢ and 5¢ stamps.

FILL IN THE BLANKS

4. Prove $1 + 3 + \cdots + (2n - 1) = n^2$. Base: $______ = ______$. Step: add the next odd number $______$: $k^2 + (2k + 1) = ______ = (k + 1)^2$. ■
5. Why does the postage proof need four base cases?

WRITE IT YOURSELF

6. Prove $2^n > n^2$ for every $n \geq 5$.
7. Prove 3 divides $n^3 - n$ for every $n \geq 0$.

SPOT THE LIE

8. "All horses are the same color" by induction. Where does it break?

CHALLENGE

9. Draw the tromino tiling of an 8×8 board missing its top-left corner using the proof's recipe.



EPISODE 5

Count Me In

5:48 · The Curve · a free EECS 203 course from Bot That Lyfe

Count the choices one step at a time, multiply, and divide out the orders that don't matter.

IN THIS EPISODE

- The product rule (outfits, passwords)
- Order matters vs. doesn't (permutations vs. combinations)
- The pigeonhole principle and the birthday problem
- Monty Hall and stars and bars

WORDS TO KNOW

Product rule

One choice then another → multiply.

$C(n, k)$

Ways to choose k from n when order doesn't matter.

Pigeonhole principle

More pigeons than holes → some hole has two.

WHY COMPUTER SCIENCE NEEDS THIS

Counting is how you size a search space: why an 8-character password resists brute force, why hash tables have collisions (pigeonhole), how many test cases an input space really has, and the birthday bound that sets the strength of hash functions in cryptography.

WALK THE PROOF

10 mana'eesh, 5 friends

1. Line up the 10 identical breads.
2. Drop in 4 dividers: before the 1st is friend 1's, between the 1st and 2nd is friend 2's, and so on.
3. 14 spots, choose which 4 are dividers: $C(14, 4) = 1001$.
4. If everyone gets one: hand out 5 first, then $C(9, 4) = 126$.

KEEP THIS TEMPLATE

COUNT IT
ordered, no repeats: $n \cdot (n-1) \cdot \dots$ (permutation)
unordered: divide by $k!$ → $C(n, k)$
 k identical → n bins: $C(k+n-1, n-1)$
more items than bins: pigeonhole!

COMMON MISTAKES

- Counting ordered when order doesn't matter ($5 \times 4 = 20$ co-captains)
- Assuming two remaining doors are 50/50
- Forgetting the leap day in the birthday pigeonhole

Sources. Rosen, K. H. Discrete Mathematics and Its Applications, §6.1–6.5 (counting, pigeonhole, permutations and combinations, distributions) · Selvin, S. (1975). A problem in probability (the Monty Hall problem). The American Statistician 29(1).

Homework: Count Me In

Work it on paper, then check the answer key at the back. Every problem is EECS 203 level.

WARM-UPS

1. 4 shirts, 3 pants, 2 pairs of shoes: how many outfits?
2. How many 4-digit PINs (0000–9999)?
3. Compute $C(5, 2)$ and list the pairs from $\{A, B, C, D, E\}$.

FILL IN THE BLANKS

4. A 3-person committee from 6: in order $6 \times 5 \times 4 = \underline{\hspace{1cm}}$; each committee counted $\underline{\hspace{1cm}}$ times; so $\underline{\hspace{1cm}}$ committees.
5. 13 people share a birth month: pigeons $\underline{\hspace{1cm}}$, holes $\underline{\hspace{1cm}}$.

WRITE IT YOURSELF

6. 12 identical cookies to 4 kids: how many ways? At least 2 each?
7. Prove that among any 5 integers, two leave the same remainder mod 4.

SPOT THE LIE

8. "After the host opens a goat door, it's 50/50, so switching doesn't matter."

CHALLENGE

9. Prove that among any 6 people there are 3 mutual friends or 3 mutual strangers.



EPISODE 6

The Curve

5:33 · The Curve · a free EECS 203 course from Bot That Lyfe

A graph is dots and lines. Count the ends of the lines and you learn what's possible.

IN THIS EPISODE

- Graphs, degree, and the handshake lemma
- Euler paths: exactly two odd corners
- Graph coloring as scheduling; why a triangle forces 3 colors
- The curve compares you to each other; the proof compares you to the truth

WORDS TO KNOW

Degree

How many edges touch a vertex.

Handshake lemma

Degrees sum to $2 \times$ edges, so the number of odd-degree vertices is even.

Chromatic number

The fewest colors so no edge joins two vertices of the same color.

WHY COMPUTER SCIENCE NEEDS THIS

Graphs are everywhere in computing: maps and routing (shortest paths), social networks, the internet itself, dependency graphs in build systems, and register allocation in compilers — which is literally graph coloring. Exam timetabling and job scheduling are coloring problems too.

WALK THE PROOF

Fewest time slots

1. Draw a vertex per session and an edge for every pair that shares students.
2. Find a triangle (Logic—Proofs—Sets): it needs 3 different colors, so at least 3 slots.
3. Color greedily: 6 PM {Logic, Induction, Graphs}, 7 PM {Proofs, Counting}, 8 PM {Sets}.
4. 3 works and 2 can't, so 3 is the fewest. ■

KEEP THIS TEMPLATE

GRAPH CHECKS

Σ degrees = $2 \times$ edges

Euler path: connected + 0 or 2 odd vertices

Lower bound on colors: biggest clique (triangle $\rightarrow$ 3)

Upper bound: show an actual coloring

COMMON MISTAKES

- Euler circuit claims on disconnected graphs
- Claiming a coloring is optimal without a lower bound
- Counting each handshake once instead of twice

Sources. Rosen, K. H. Discrete Mathematics and Its Applications, §10.1–10.5, §10.8 (graphs, Euler paths, coloring), §11.1 (trees) · Euler, L. (1736). Solutio problematis ad geometriam situs pertinentis (the bridges of Königsberg).

Homework: The Curve

Work it on paper, then check the answer key at the back. Every problem is EECS 203 level.

WARM-UPS

1. Degrees 3, 3, 2, 2, 2: how many edges?
2. Can a party have exactly one person who shook an odd number of hands?
3. 4 dots, each connected to every other: how many edges?

FILL IN THE BLANKS

4. Degrees sum to $2 \times$ edges because each edge has ____ ends, so the number of odd-degree vertices is ____.
5. Euler path? (2, 2, 2, 2): ____ · (3, 3, 2, 2): ____ · (3, 3, 3, 3): ____ (connected graphs).

WRITE IT YOURSELF

6. Prove by induction that every tree with n vertices has $n - 1$ edges.
7. Why can't a 5-cycle be 2-colored? Color it with 3.

SPOT THE LIE

8. "Every vertex has even degree, so there's an Euler circuit." When is that false?

CHALLENGE

9. Exams A–E, conflicts A–B, A–C, B–C, C–D, D–E. Find the fewest slots and prove it.

The season on one page

EPISODE 1

Knights and Knaves

An if-then is a promise. It's only broken when you do your part and they don't do theirs.

EPISODE 3

Some Infinities Are Bigger

Two sets are the same size when you can pair them up perfectly, with nobody left out.

EPISODE 5

Count Me In

Count the choices one step at a time, multiply, and divide out the orders that don't matter.

EPISODE 2

Proof by Contradiction

A proof is an argument nobody can wiggle out of. Checking examples is a **feeling**, not a proof.

EPISODE 4

The Next Step

Induction: get on the first step, and prove that from any step you can reach the next one.

EPISODE 6

The Curve

A graph is dots and lines. Count the ends of the lines and you learn what's possible.

THE PROOF FRAMEWORK

THE PROOF FRAMEWORK

GIVEN: what do we know? (write the definitions out)
PROVE: exactly what must be shown – for every case
STRATEGY: direct · contrapositive · contradiction · cases · induction
DERIVATION: every line + its reason
■ did we prove the original claim, every case?

PROOF BY CONTRADICTION

PROOF BY CONTRADICTION

CLAIM: _____
PRETEND: the opposite is true: ____
FOLLOW: step → step → step (reasons)
BREAKS: ✗ ____ contradicts ____
SO: the pretend was wrong. ■

KEEP GOING

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nate@autonateai.com

Answer key

Episode 1 · Knights and Knaves

1. (a) proposition, true (b) not one — a command (c) not one — depends on x (d) proposition, false (9).
2. Converse: "if the Diag is wet, it's raining" (no — sprinklers). Contrapositive: "if the Diag isn't wet, it isn't raining." The contrapositive means the same.
3. T, F, T, T for rows TT, TF, FT, FF. False only when p is true and q is false.
4. $\neg p$: F F T T · $\neg q$: F T F T · $p \rightarrow q$: T F T T · $\neg q \rightarrow \neg p$: T F T T. Identical, so equivalent.
5. true · knave · false · knight.
6. If A were a knave, the sentence would be false, so neither is a knave — so A is a knight. Contradiction. So A is a knight; the sentence is true; A isn't a knave, so B is. ■
7. (a) "At least one student did not fail." $\exists s \neg F(s)$. (b) "No student got a 100." $\forall s \neg H(s)$.
8. It's the converse — Jaylen could have passed without studying. Valid: "If you study you'll pass; Jaylen didn't pass; so he didn't study."
9. If A were a knight, B would be a knight saying a falsehood ("A is a knave"). So A is a knave, B's sentence is true so B is a knight, and C's sentence is false so C is a knave. Check: A's claim is false, as it should be. ■
10. Check "inside" and "no ticket" — the same shape as A and 7 (check p and not-q). It feels easier because you already know how tickets work.

Episode 2 · Proof by Contradiction

1. $13 = 2 \cdot 6 + 1$, $k = 6$. $-5 = 2 \cdot (-3) + 1$, $k = -3$.
2. A "for every" claim can fail at an unchecked number ($n^2 + n + 41$ fails at 40). Only an argument covering every n is a proof.
3. 3, 0, 1.
4. $2a + 2b = 2(a + b)$.
5. even · even · $4k^2 \cdot 2k^2$.
6. Contrapositive: n even $\rightarrow 3n + 2$ even. $n = 2k$ gives $6k + 2 = 2(3k + 1)$. ■
7. $\sqrt{3} = a/b$ lowest terms $\rightarrow a^2 = 3b^2 \rightarrow 3 \mid a^2 \rightarrow 3 \mid a$ (a non-multiple of 3 squares to remainder 1) $\rightarrow a = 3c \rightarrow b^2 = 3c^2 \rightarrow 3 \mid b$. Contradiction. ■
8. Dividing by $a - b$, which is 0.
9. $n^2 + n = n(n + 1)$; one of n , $n + 1$ is even, so the product is even. ■
10. Every square leaves remainder 0 or 1 mod 4, never 2. ■

Episode 3 · Some Infinities Are Bigger

1. {1, 2, 3, 4, 5}; {3, 4}; {1, 2}.
2. Yes.
3. Not one-to-one ($a, b \rightarrow 1$); not onto (2 missed).
4. $2n + 1 \cdot n$. Not onto: it never hits an even number.
5. $2n \cdot k$.
6. 0, 1, -1, 2, -2, 3, -3. Positive m comes only from $2m - 1$, negative -m only from $2m$, 0 only from 0.
7. Stool $n \rightarrow 2n$; the odd stools empty; new customer k takes stool $2k - 1$.
8. Finite intuition. $n \mapsto 2n$ is a perfect pairing, so they're the same size.
9. Subset $S \leftrightarrow$ the 0/1 string with 1 in spot n exactly when $n \in S$. That's a bijection with infinite 0/1 strings, which are uncountable. ■

Episode 4 · The Next Step

1. 55.
2. Getting on the first step is the base; "from step k you reach $k + 1$ " is the step.
3. $4 + 4 + 4 + 5 + 5$.
4. $1 \cdot 1 \cdot 2k + 1 \cdot k^2 + 2k + 1$.
5. The step reaches back 4 (n uses $n - 4$), so 12–15 must be checked by hand.
6. Base $32 > 25$. Step: $2^{k+1} = 2 \cdot 2^k > 2k^2 \geq (k + 1)^2$, since $k^2 \geq 2k + 1$ for $k \geq 3$. ■
7. Base 0. Step: $(k + 1)^3 - (k + 1) = (k^3 - k) + 3(k^2 + k)$. ■
8. From 1 horse to 2: the two groups of 1 don't overlap, so nothing forces them to match.
9. Middle L for the whole board (bites the three quarters without the hole), then a middle L in each 4×4 , then one L per 2×2 : 21 trominoes.

Episode 5 · Count Me In

1. 24.
2. 10,000.
3. 10: AB AC AD AE BC BD BE CD CE DE.
4. $120 \cdot 6 \cdot 20$.
5. $13 \cdot 12$.
6. $C(15, 3) = 455$. At least 2: give 8 first, $C(7, 3) = 35$.
7. Only 4 remainders, 5 integers $\rightarrow$ pigeonhole. ■
8. Your first pick stays right $1/3$ of the time; the host always shows a goat, so switching wins $2/3$.
9. Pick P. Of the other 5, at least 3 are friends with P or 3 are strangers (pigeonhole). If 3 friends A, B, C: any friendly pair among them plus P makes 3 mutual friends; otherwise A, B, C are mutual strangers. ■

Episode 6 · The Curve

1. 6.
2. No — the count of odd-degree people is always even.
3. 6.
4. two · even.
5. Euler circuit · path between the two degree-3 vertices · none.
6. Base: 1 vertex, 0 edges. Step: remove a leaf and its edge $\rightarrow$ a k -vertex tree with $k - 1$ edges; add it back: k edges. ■
7. Colors must alternate around the cycle; with 5 vertices the last touches the first. 3 colors: 1, 2, 1, 2, 3.
8. When the graph isn't connected (two separate triangles).
9. Triangle A–B–C forces 3. A1, B2, C3, D1, E2 works. Fewest: 3. ■